Final Exam

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Exercise 1. (5 points) Find the maxima and minima of the following function, subject to the corresponding constraint:

$$f(x_1, x_2) = (x_1 - 1)^2 + (x_2 + 1)^2 \qquad \textit{subject to} \quad x_1^2 + x_2^2 = 2$$

Exercise 2. (5 points) Solve the following differential equation with separable variables:

$$x'(t) = (t+1)[x^2(t) + x(t)]$$

Exercise 3. (5 points) Solve the following linear first-order differential equation:

$$x'(t) = e^t x(t) + e^{2t}$$

Exercise 4. (5 points) Solve the following Cauchy problem:

$$\begin{cases} x'(t) = 3te^{t^2} x(t) \\ x(0) = 1 \end{cases}$$

Exercise 5. (5 points) Solve the following linear first-order differential system with constant coefficients:

$$\begin{cases} x_1'(t) = x_3(t) \\ x_2'(t) = 3x_1(t) + 7x_2(t) - 9x_3(t) \\ x_3'(t) = 2x_2 - x_3(t) \end{cases}$$

Exercise 6. (5 points) Solve the following linear second order differential equation with constant coefficients transforming it into a first-order linear system (check your solution with the characteristic function):

$$x''(t) - 5x'(t) + 6x(t) = 7$$